Asynchronous Verifiable Secret Sharing in Optimal Communication Complexity

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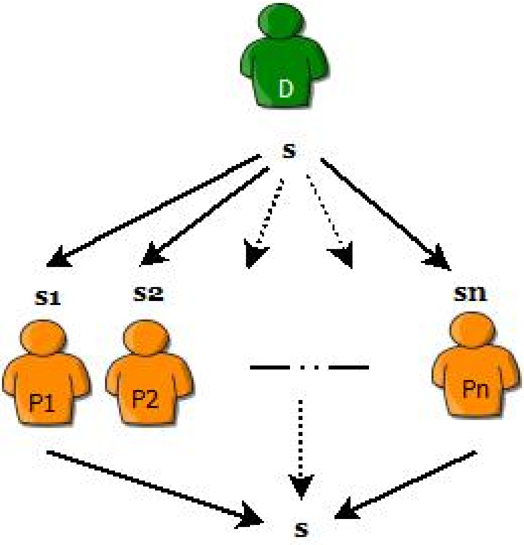
Outline

- Background: Verifiable Secret Sharing (VSS)
- Asynchronous Verifiable Secret Sharing (AVSS)
- State-of-the-art Protocols
- What we want to Achieve and How
- Our Protocols

An (n,t)-VSS: Sharing and Reconstruction



Asynchronous System Model

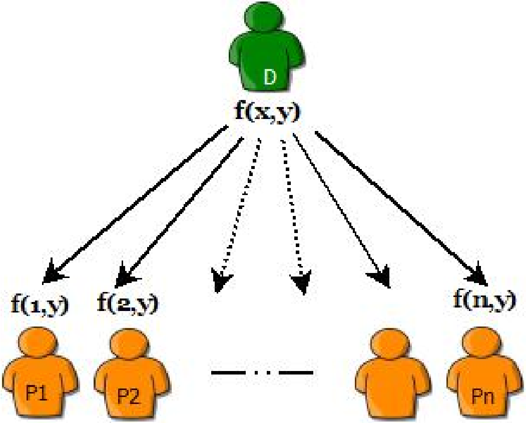
The Adversary:

- Controls the network and may delay messages between any two honest parties
- Cannot read or modify these messages
- Has to eventually deliver all the messages by honest parties
- Can corrupt at most t parties, out of n

In this setting, the optimal resiliency bound is $n \geq 3t + 1$

State-of-the-art Protocol for AVSS

Sharing Phase:



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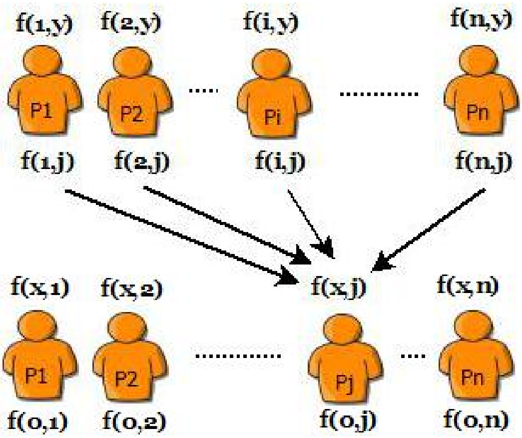

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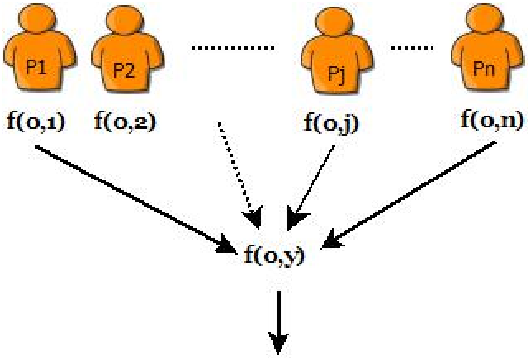
Reconstruction Phase:



Compute the secret $s = f(0, 0)$

State-of-the-art Protocol for AVSS

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Commitment matrix $\mathbf{C} = \{C_{jl}\} = g^{f_{jl}}$

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Message complexity: $O(n^2)$,

Communication complexity: $O(\kappa n^3)$,

where κ is the security parameter

Reference : C. Cachin, K. Kursawe, A.Lysyanskaya, and R. Strobl. Asynchronous Verifiable Secret Sharing and Proactive Cryptosystems, ACM CCS'02.

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Solution : Polynomial Commitments

Helps commit to a vector by publishing just one value

Reference : A.Kate, G. M. Zaverucha, and I. Goldberg. Constant-Size Commitments to Polynomials and Their Applications. In Proceedings of ASIACRYPT'10.

A PolyCommit scheme

Setup$(1^\kappa, t)$ generates an appropriate algebraic structure $\mathcal{G} = \langle e, \mathbb{G}, \mathbb{G}_T \rangle$ and the system parameter PK

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VerifyEval$(PK, \mathcal{C}, i, \phi(i), w_i)$ verifies that $\phi(i)$ is indeed the evaluation of the polynomial committed in $\mathcal{C}$

Our Protocol

Protocol for AVSS - 1

Dealer

- D selects a polynomial $\phi(x)$, such that $\phi(0) = s$.
- $\mathcal{C} = \text{Commit}(PK, \phi(x))$, $w_i = \text{CreateWitness}((PK, \phi(x), i))$
- D sends ($\mathcal{C}$, w_i, $\phi(i)$) to every party P_i.

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Party P_i

- If VerifyEval(PK,$\mathcal{C}$, i, $\phi(i)$, w_i) succeeds, send (`echo`, $\mathcal{C}$)
- On receiving $(n - t)$ (`echo`, $\mathcal{C}$), send (`ready`, `holder`, $\mathcal{C}$)
- Otherwise:
  - (a) On receiving $(n - 2t)$ (`ready`,*, $\mathcal{C}$) signals, send (`ready`, `holder`, $\mathcal{C}$) to every party P_j.
  - (b) On receiving $(n - 2t)$ (`ready`,*, $\mathcal{C}'$) signals, send (`ready`, `non-holder`, $\mathcal{C}'$) to every party P_j.
- On receiving $(n - t)$ (`ready`, $\mathcal{C}$) signals, and at least $(n - 2t)$ contain `holder`, terminate.

Salient Points

- There are at least $n - 2t \geq 3t + 1 - 2t = t + 1$ honest parties with correct shares
- There are at most n `send`, n^2 `echo` and n^2 `ready` messages

Properties of AVSS

Liveness. If the dealer D is honest, then all honest parties complete sharing.

Secrecy. If D is honest, then the adversary has no information about s.

Agreement. If some honest party completes the sharing phase, then all honest parties will eventually complete the sharing phase.

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Correctness. Once all honest parties complete sharing, there exists a fixed value $z \in \mathbb{Z}_p$, such that the following holds:

(a) If an honest dealer has shared the secret s, then $s = z$.

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Liveness. If the dealer D is honest, then all honest parties complete sharing.

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Strong Correctness. Once $t+1$ honest parties complete sharing, there exists a fixed value $z \in \mathbb{Z}_p$, such that the following holds:

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Protocol for AVSS-SC

- Dealer `sends` polynomials $\phi^0(x), \phi^1(x), ...\phi^n(x)$, with $\phi^k(x) = F(x, k)$, $F(x, y)$ is of degree $\leq t$. Commitments: $\mathcal{C}^0, \mathcal{C}^1, ..., \mathcal{C}^n$.

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- There will be at least $t + 1$ honest parties with correct polynomials $\phi^k(x) = F(x, k)$, and they compute their shares $s_k = \phi^k(0) = F(0, k) = F(k, 0) = \phi^0(k)$

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- These $t + 1$ parties can enable any P_i to reconstruct its polynomial $\phi^i(x)$ by sending $\phi^k(i) = \phi^i(k)$

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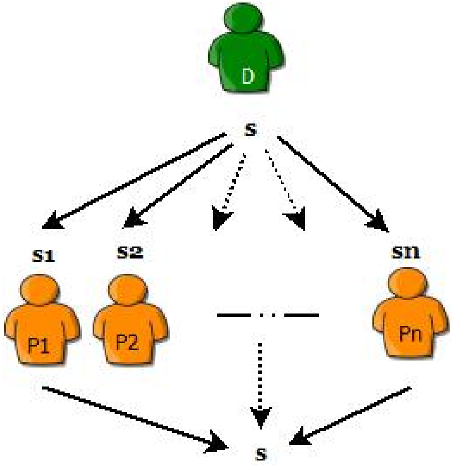
Solution: Perform another round of PolyCommit on hash values of the commitments.

Complexity Analysis

Theorem

A protocol for AVSS is sufficient to generate a protocol for a reliable broadcast.

Complexity Analysis



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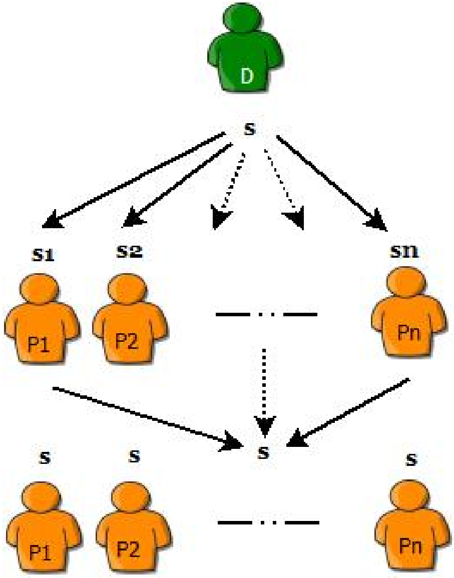

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Theorem

If a reliable broadcast protocol terminates, the number of messages exchanged is lower bounded by $max\{(n-t),(1+t/2)^2\}$.

Reference : D. Dolev and R. Reischuk. Bounds on information exchange for byzantine agreement. J. ACM, 1985

Contributions

- Incorporation of Polynomial Commitments to solve AVSS with improved complexity
- This protocol for AVSS achieves optimal complexity

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Thank You